**SUBJECT:MATHEMATICS**

**CLASS: SS2**

**TOPIC: SEQUENCES AND SERIES**

 SEQUENCE

These are groups of things that are arranged in an order especially following one another in time.

Thus, Sequence is an ordered set of mathematical quantities called terms.

 EXAMPLES

1. 1, 2, 3, 4, 5, …
2. 2 , 4, 6, 8, ….
3. 2,6,18,58

 FINITE SEQUENCE

Finite Sequence is one whose number of terms is known or which can be determined.

Eg; 3, 5, 9, 12,…, n

The last number is known.

 Examples

1. Find the terms of the sequence defined by Tn = $n^{2}$ + 2n – 1

Solution :

We shall consider n=1,2,3,4

In Tn=$n^{2}$+2n-1

When n=1

 T1=$1^{2}$+2(1)-1

 =1+2-1

 =3-1

 =2

When n =2

 =$2^{2}$ + 2(2)-1

 =4+4-1

 =7

When n=3

 =$3^{2}$+2(3)-1

 =9+6-1

 =14

When n=4

 =$4^{2}$+2(4)-1

 =16+8-1

 =23

1. Find the first five terms of the sequence $2^{n}$+$n^{2}$

Solution

Let n=1,2,3,4,5

When n=1

$2^{n+}n^{2}$ will become

$2^{1}$+$1^{2}$ =2+1=3

When n=2

$2^{2}$+$2^{2}$ =4+4=8

When n=3

$2^{3}$+$3^{2}$ =8+9=17

When n=4

$2^{4}$+$4^{2}$=16+16=32

When n=5

$2^{5}$+$5^{2}$=32+25=57

First five terms =3,8,17,32,57

 SERIES

Series is obtained by forming the sum of the terms of a sequence.

 ARITHMETIC PROGRESSION

Arithmetic progression is a form of sequence in which each term is gotten by the addition of a common difference to a preceding one. A.p’s are sequence that follow simple addition rule.

 Nth term of an AP

We denote the first term by “a” and the common difference by “d”.

Let n represent the nth term.

 Generally, nth term of an Ap→ Tn =a+(n-1)d.

 Examples

1) find the 8th term of an AP 3,6,9,…

 Solution

Tn = a+(n-1)d

a=3, n=8,d=6-3=8

substituting, we have

T8 =3+(8-1)3

 =3+7×3

 =3+21=24

2)If the 11th term of an AP is 42 and its first term is 2,find the common difference .

 Solution

Tn=a+(n-1)d

Tn=42, a=2, d=?, n=11

Substituting,

42=2+(11-1)d

42=2+10d

42-2=10d

d=40/10 =4

3) The first term of an AP is to twice the common difference d.Find in terms of d,the 5th termof the AP.

 Solution

a=2d

Tn=a+(n-1)d

but a=2d. n=5

T5=2d+(5-1)d

 = 2d + 4d=6d

4) What is the 25th term of 5, 9, 13,…

 Solution

Critical look at the sequence shows that it has the properties of an AP, which is the addition of the common difference to each term to get the next.

Tn=a+(n-1)d

Tn=25,a=5,d=9-5=4

T25=5+(25-1)4

 =5+(24)4

 =5+96=101

5) Find the 19th term an AP, $\frac{5}{6}$, $\frac{8}{6}$, $\frac{11}{6}$, …

 Solution

Tn=a+(n-1)d

n=19,a=$\frac{5}{6}$ , d=$\frac{8}{6}$ ̶ $\frac{5}{6}$ = $\frac{3}{6}$

Substituting

Tn=$\frac{5}{6}$ + (9-1)$\frac{3}{6}$

 = $\frac{5}{6}$ + (18)$ \frac{3}{6}$

 = $\frac{5}{6}$ + 9

 = $\frac{59}{6}$

 =9 $\frac{5}{6}$

 ARITHMETIC MEAN

By the arithmetic mean of two numbers, we mean another number which when placed between the two numbers gives two consecutive terms of an AP.

 Example

Find the arithmetic mean of

i) 11 and 51 (ii) -13 and 15

Solution

i) AM = $\frac{11+51}{2}$ = $\frac{62}{2}$ = 31

ii) AM = $\frac{-13+15}{2}$ = $\frac{2}{2}$ = 1

 SUM OF AN AP

The sum an AP refers to the addition of all the terms in the given AP. It is usually denoted by Sn, where n is the progression.

:. Sn = $\frac{n}{2 }$[2a+(n-1)d]

A case where the first term (a) and last term only are given ;

Sn= $\frac{n}{2}$(a+l)

 Examples

1) Find the sum of the first 30 terms of the AP 3,5,7…

Solution:

Sn = $\frac{n}{2 }$[2a+(n-1)d]

n=30, a=3, d=2

Substituting,

$\frac{30}{2}$[2×3 + (30-1)2]

15[6+(29)2]

15(6+58)

15(64)=960

2) The sum of the first 9 terms of an AP is 72 and the sum of the next 4 terms is 71. Find the AP

Solution:

 Sn = $\frac{n}{2 }$[2a+(n-1)d]

n=9,a=? d=?

72= $\frac{9}{2}$[2a+(9-1)d]

72×2=9(2a+8d)

144=9(2a+8d)

Divide both side by 9

16=2a+8d ------(1)

The next four terms from 9th term is 13th term ,thus,

S13 =S9+Snext four terms

 =72+71=143

S13=143=$\frac{13}{2}$[2a+(13-1)d]

143×2=13(2a + 12d)

Divide both side by 13

22=2a + 12d -----(2)

Solving equation 1 and 2

 2a +8d=16

-(2a +12d=22)

 4d =-6

 d = $\frac{3}{2}$

substituting d value into equation 1

2a + 8($\frac{3}{2}$) = 16

2a + 12 =16

2a=16-12

2a=4

a=2

The AP is 2,3$\frac{1}{2}$,5…

3) In an AP, the first term is 2 and the sum of the 1st and 6th term is 16$\frac{1}{2}$ , what is the 4th term

Solution:

T1 +T6=16$\frac{1}{2}$

T1=a, T6=a + (6-1)d=a+5d

T1 +T6=a+a+5d=16$\frac{1}{2}$

 2a+5d=16$\frac{1}{2}$

but a=2

2×2+5d=16$\frac{1}{2}$

4+5d=16$\frac{1}{2}$

5d=16$\frac{1}{2}$ ̶ 4

5d=$12\frac{1}{2}$

d=$\frac{25}{2}$÷5

d=2$\frac{1}{2}$

thus,T4=2+(4-1) 2$\frac{1}{2}$

T4 =2+(3)$\frac{5}{2}$ = $\frac{19}{2}$ =9$\frac{1}{2}$

 ASSIGNMENT

1) The sum of the 1st and 2nd term of an AP is 4 and the 10th term is 19. Find nthe sum of the 5th and 6th terms.

2) The first term of an AP is 3 and fifth term is 9. Find the number of terms in the progression if the sum is 81.

**WEEK SIX**

**TOPIC: GEOMETRIC PROGRESSION**

A geometric progression (GP) is a form sequence, which has a common ratio between any of the term and its preceding term.

**Examples**

i) 5, 15, 45,135 …

ii) 640,320,160 …

**Solution**

The first term a =5, the common ratio, that is the constant with which the terms are multiplied with to get the next term, r=$\frac{15}{5}$ =$\frac{45}{15}$ =$\frac{135}{45 }$ =3

ii) a = 640 , r = $\frac{320}{640}$ = $\frac{1}{2}$

**Nth term of a GP**

We denoted the first term by ‘a’ and the common ratio by ‘r’.

In general, for any given GP a, ar, ar2 … arn

Tn =ar n-1

**Examples**

1. Find the 6th term of the GP, 3, 9, 27…

**Solution**

Tn =arn-1

N=6, a=3, r=3

T6= 3×36-1

= 3 × 35

= 3 × 243 = 729

1. The fourth term of a GP is 1. If its first term is 64, find the common ratio.

**Solution**

T4 =ar4-1

But a= 64, T4 =1

Thus, 1=64r3

Divide both sides by 64

[${1}/{64}$]1/3 =r3

Taking the cube root of both sides

[${1}/{64}$]1/3= (r3)1/3

${1}/{4 }$= r

1. The first term of a geometric progression (GP) is x and the third term is y. find an expression for the common ratio in terms of x and y.

**Solution**

Tn = arn-1

 n = 3, a = x, T3 = y, r =?

 y = xr3-1

 y = xr2

 Divide both sides by x

 $\frac{y}{x}$ = r2

Square root both side to clear the square on r

 $\sqrt{y/x}$ = r

1. If $ \frac{16}{9}$ , x, 1, y are in geometric progression (GP), find the product of x and y.

**Solution**

3rd term (T3) =1, a = $\frac{16}{9}$

Using T3 to get the common ratio while applying

Tn = arn-1

T3 =1 = $\frac{16}{9 }$ (r)3-1

1 = $\frac{16}{9}$r2

1 $×$ 9 = 16r2

$\frac{9}{16}$ = r2

${[3}/{4}]$2 = r2

r=$^{3}/\_{4}$

x is second term

ar = $^{16}/\_{9}$ × $^{3}/\_{4}$

 = $^{4}/\_{3}$

Y is the 4th term

ar3 = $^{16}/\_{9}$ × [${3}/{4}$]3

 =$^{16}/\_{9}$ $× ^{27}/\_{64}$

Y =$^{3}/\_{4}$

:. , xy = $^{4}/\_{3 ×}$ $^{3}/\_{4}$ = 1

**SUM OF GEOMETRIC PROGRESSION**

The sum, Sn of n terms of a GP is Sn = $\frac{a(1-r^{n})}{1-r}$

This formula holds when r < 1

Sn = $\frac{a(r^{n}-i)}{r-1}$

This formula is used when r>1. Usage of either formula one or two depends only on the common ratio (r) of the Geometric progression given.

Examples

1. Find the sum of the first 6 terms of the GP 10, 20, 40…

Solution

Check for the GP’s common ratio,

r = $\frac{20}{10}$ or $\frac{40}{20}$

 Sn = $\frac{a(r^{n}-1)}{r-1}$

 Sn = $\frac{10( 2^{6}- 1 )}{2-1}$

 = 10(64 – 1)

 = 10 × 63 = 630

1. If the second and fourth terms of a GP are 8 and 32 respectively. What is the sum of the first four terms?

Solution

We need to know a and r before finding sum. Thus,

Tn = arn-1

T2 = 8 = ar2-1

 8 = ar ------ (1)

T4 = 32 = ar4-1

 32 = ar3------ (2

Dividing (2) by (1))

$\frac{ar^{3}}{ar}$ = $\frac{32}{8}$

r2 = 4

r = $\sqrt{4}$

r = 2

Put r = 2 into (1)

ar = 8 will become

2a = 8

a = 4

but common ratio r = 2 >1 thus,

 Sn = $\frac{a(r^{2}-1)}{r-1}$

 S4 = $\frac{4(2^{4}-1)}{2-1}$

 = 4 (16 - 1)

 = 4 × 15 = 60

1. If the 2nd and 5th term of a GP is -6 and 48 respectively, find the sum of the first four terms.

Solution:

We need to find a and r

T2 =-6 = ar ----- (1)

T5 = 48 = ar4 ----- (2)

Divide (2) by (1)

$\frac{ar^{4}}{ar}$ = $\frac{48}{-6}$

r3 = -8

r3 = (-2)3

r = -2

put r = -2 into (1)

ar = -6 will become

-2a = -6

a = $\frac{-6}{-2}$ = 3

since our common ratio r -2<1.

We use formula is Sn = $\frac{a(1-r^{n})}{1-r}$

S4 = $\frac{3 [1-(-2)^{4}\})}{1-(-2)}$

since the power of -2 is even ,therefore, the result is positive.

 S4  **=** $\frac{3 (1-16 )}{1+2 }$ = $\frac{3 (-15)}{3}$ = -15

 **SUM TO INFINITY (S∞)**

A GP whose common ratio is between -1 and +1 say $\frac{-1}{2}$ , $\frac{1}{2}$ $, \frac{-1}{4}$ , $\frac{1}{4}$ etc has a sum which approaches infinity. It is given as S∞ = $\frac{a}{1-r}$

Examples

1. The sum to infinity of the GP 4,2,1,…

Solution

 S∞ = $\frac{a}{1-r}$

a = 4 , r = $^{1}/\_{2}$

S= $\frac{4}{1-^{1}/\_{2}}$ = 4 ÷ $^{1}/\_{2}$ = 8

1. Two geometric progression (G.PS) have equal sums to infinity. Their first terms are 80 and 25 respectively . if the common ratio of the first is $^{4}/\_{5}$ , find the common ratio of the second.

Solution

For the 1st S∞ = $\frac{80}{1-^{4}/\_{5}}$

= 80 × $^{1}/\_{5}$ = 80 × 5 = 400

Since their sum to infinity is the same for the 2nd GP.

400 = $\frac{25}{1-r}$

1. 1-r) = 25

 1 – r = $ \frac{ 25}{400}$

 1 – r = $\frac{1}{16}$

 r = 1 - $^{1}/\_{16}$

r = $^{15}/\_{16}$

ASSIGNMENT

1. Find the of the first five terms of GP 2,6,18,…
2. Find the sum of 30 terms of the GP , 15,45,135,…

**WEEK EIGHT:**

**SUBJECT: MATHEMATICS**

**CLASS: SS2**

**TOPIC: QUADRATIC EQUATIONS**

**COMPLETING THE SQUARE METHOD**

 Let’s recall that a quadratic equation which is factorizable can be solved by factorization method. Where such equation is not factorizable, we may resort to the method of completing the square. This requires a person to make a quadratic expression a perfect square.

 9 is a perfect square because there is a number 3, whose square is 9. In the same way, X2+8x+16 is a perfect square because there is an algebraic number, x+4 whose square is X2+8x+16. That is, (x+4)2 = X2+8x+16.

On the other hand, X2+8x is not a perfect square but we can find a number, K, which when added to it, would make it a perfect square.

The process of finding this number is known as “**Completing the square**” and it can be used in solving quadratic equations.

**STEPS ON HOW TO SOLVE QUADRATIC EQUATIONS USING COMPLETING THE SQUARE METHOD**

General form of a quadratic equation ax2 +bx +c =0

1. Divide through by the coefficient of x2 to make the coefficient of x2 unity.
2. Carry the constant to the right hand side (RHS)of the equation
3. Add the square of half the coefficient of x to both sides.
4. On the left hand side (LHS), we take two terms under squares while we perform arithmetic operation on Right hand side (RHS)
5. Take the square root of both sides.
6. Solve the arithmetic.

**Examples**

1. Using the method of completing the square, find the root of the equation: X2- 6x + 7 =0, correct to one decimal place.

Solution:

X2-6x+7=0

Carry the constant to the right hand side of the equation

X2-6x=-7

 Add the square of half the coefficient of x to both sides.

 X2-6x+ (-3)2 = -7+ (-3)2

At the left hand side (LHS), select the one with squares.

(X-3)2 = -7 + 9

(X-3)2 = 2

Take square root of both sides

X-3 = ±$\sqrt{2}$

X= 3 ± 1.414

 = 3 + 1.414 OR 3 – 1.414

 = 4.4 OR 1.6 to 1 d.p.

1. Solve X2-4x-12=0 using completing the square method.

Solution

X2-4x-12=0

X2-4x=12

X2-4x+ (-2)2=12+ (-2)2

(X-2)2= 12+4

(X-2)2=16

X-2=±$\sqrt{16}$

X= 2 ± 4

X= 2+4 or 2-4

X = 6 or -2

1. Using completing the square method solve; 2X2-7X+6=0

Solution

2X2-7X+6=0

X2-$ \frac{7X}{2} $+3=0

X2 - $\frac{7X}{2}$ = -3

X2 - $\frac{7X}{2}$ +$\left(-\frac{7}{4}\right)$2= -3 +$\left(-\frac{7}{4}\right)$2

$\left(X-\frac{7}{4}\right)$2 = -3 + $\frac{49}{16}$

Finding the LCM of the RHS (Right hand side)

$\left(X-\frac{7}{4}\right)$2 = $\frac{-48+49}{16}$

$\left(X-\frac{7}{4}\right)$2 = $\frac{1}{16}$

$X$ - $\frac{7}{4}$ = ±$\sqrt{\frac{1}{16}}$

$X$ - $\frac{7}{4}$ = ±$ \frac{1}{4}$

$X$ = $\frac{7}{4}$ + $\frac{1}{4}$ OR = $\frac{7}{4}$ - $\frac{1}{4}$

 = $\frac{8}{4}$ OR $\frac{6}{4}$

 = 2 OR $\frac{3}{2}$

**MAKING QUADRATIC EXPRESSION PERFECT SQUARE BY ADDING A CONSTANT K**

**Examples**

1. What should be added to the following to make them perfect squares
2. X2+4x b) X 2 – 6x c) 4X2 – 12x

Solution

For each of them, write down the factor of the perfect square formed by adding the quantities.

1. X2+4x

The required number is the square of half of the coefficient of X which is +4.

$\frac{4}{2}$ = 2, (2)2 = 4

$Therefore,$ The perfect square is X2+4x+4 which will factorize into (x+2)2

1. X 2 – 6x

 $-\frac{6}{2}$ = -3, (-3)2 = 9

= X2 – 6x + 9

1. 4X2 – 12x

Divide through by the coefficient of X2 which is 4

$\frac{4x^{2}}{4}$ - $\frac{12x}{4}$ = X2 – 3x

We then proceed as before,

$-\frac{3}{2}$, $\left(-\frac{3}{2}\right)$2 = $\frac{9}{4}$

This means that X2 – 3x + $\frac{9}{4}$ is a perfect square.

We then multiply all through by 4 to get 4X2 – 12x + 9.

Therefore the number to be added to 4X2 – 12x to make it a perfect square is 9.

**WEEK 9**

**SUBJECT: MATHEMATICS**

**TOPIC: CONSTRUCTION OF QUADRATIC EQUATION FROM SUM AND PRODUCTS OF ROOTS**

An equation can be formed if its roots are given.

**Properties**

1. The coefficient of x2 is unity.
2. The sum of the roots is the coefficient of x with the sign changed.
3. The product of the roots is the constant term.

Thus, if the roots of a quadratic equation are given, then the equation can be obtained (with the usual three terms) as

 X2 – (sum of roots) x + (products of roots) =0

**Examples**

1. Find a quadratic equation whose roots are -4 and 7

**Solution**

The given roots are -4 and 7

* x=-4 or x=7

x+4 =0 or x-7=0

Since either of the two factors or both is equal to zero, it means that their product must be zero.

(x+4)(x-7) = 0

Expanding

X2 – 7x + 4x – 28 =0

X2 – 3x – 28 =0

1. find a quadratic equation whose roots are 3 and -7

**Solution**

* **x=3 or x=**-7

x- 3 =0 or x+7=0

(x- 3)(x+7) =0

X2+7x-3x-21=0

X2+4x-21=0

**WORD PROBLEMS LEADING TO QUADRATIC EQUATIONS**

**Examples**

1. Two numbers differ by 3 and their product is 28.find the numbers.

**Solution**

Let one of the numbers be x and the other (x-3).

Their product is 28

* X(x-3) =28

X2-3x=28

X2-3x-28=0

Factorizing

X2-7x+4x-28=0

X(x-7) +4(x-7) =0

(x-7)(x+4)=0

x-7=0 or x+4=0

x=7 or x=-4

Since the second number is (x-3)

 Therefore, the second number will be

 When x=7, (x-3) will be 7-3=4

 When x=- 4, (x-3) will be -4-3=-7

Therefore the numbers will be either 7 and 4 or -4 and -7

 **ASSIGNMENT**

Find the quadratic equation whose root

a) -$^{1}/\_{3}$ and -5 b) 5 and 6

**WEEK 10**

**SUBJECT: MATHEMATICS**

**CLASS: SS2**

**TOPIC: SIMULTANEOUS LINEAR AND QUADRATIC EQUATION**

 **Revision on Simultaneous Linear Equation**

The highest degree of the unknown in simultaneous linear equation is one. Though, in this case there are two different unknowns unlike in the simple equation, which has only one unknown.

Recall, that simultaneous linear equation can be solve either by elimination or substitution method. Though a third method of solving exists, this is graphical solution.

**Examples**

1. Solve the equations

X + 2y = 5 and X + 3y = 8

Solution:

X + 2y = 5 ------- (1)

X + 3y = 8 ------- (2)

Eliminate X to get the value of y by subtracting equation (2) from equation (1)

 X + 2y = 5

-(X + 3y= 8)

 - y =-3

Divide both sides by the coefficient of y, which is (-1)

:. Y= 3

To get the value of X, put the value of y=3 into any of the equation, [putting y=3 into equation (1)]

X + 2y = 5 becomes

X + 2(3) = 5

X + 6 = 5

Subtract 6 from both sides

X + 6 – 6 = 5 – 6

X=- 1

**Using Substitution method**

X + 2y = 5 ------- (1)

X + 3y = 8 ------- (2)

Make either X or y the subject from any of the equation

From equation (1), make X the subject

X + 2y = 5 will become

X = 5 – 2y -------(3)

Substitute the value of X into equation (2)

X + 3y = 8 will become

5 – 2y + 3y = 8

5 + y = 8

Subtract 5 from both sides to make y the subject

5 -5 + y = 8 – 5

Y = 3

Substituting y =3 into equation (1)

X + 2y = 5 will become

X + 2(3) = 5

X + 6 = 5

Subtract 6 from both sides

X + 6 -6 = 5 – 6

X = - 1

**SOLUTION OF LINEAR AND QUADRATIC EQUATION**

**Examples**

1. Solve the equations

X – y = 2 and X2 + y2 = 52

Solution

X – y = 2 ------(1)

 X2 + y2 = 52-----(2)

From (1) X – y =2

 X = y + 2

Substitute X value into (2)

X2 + y2 = 52 will become

(Y + 2)2 + y2 = 52

Y2 + 4Y + 4 + y2 = 52

2Y2 + 4Y + 4 – 52 = 0

2Y2 + 4Y – 48 = 0

Reducing the terms, we have

Y2 + 2Y – 24 = 0

Factorizing;

Y2 + 6y – 4y -24 = 0

Y (y+ 6) – 4 (y + 6) = 0

(Y + 6) (Y – 4 )=0

Y + 6 =0 or Y – 4 =0

Y= - 6 or 4

Substituting Y values into (1)

X – Y = 2 will become

X – (- 6) = 2 or X – 4 =2

X + 6 = 2 or X – 4 = 2

X = 2 – 6 or X = 2+6

X = - 4 or 8

The solution sets are (- 4 , - 6 ) and ( 8, 4) in (x,y) format

1. If Y = X2 – 4x – 10 and Y = 2. Find the values of x that satisfies both relations.

**Solution;**

Since Y = X2 – 4x – 10 and Y = 2

 Y => X2 – 4x – 10 = 2

 X2 – 4X – 10 – 2 = 0

 X2 – 4x – 12 = 0

Factorizing;

X2 – 6x + 2x – 12 = 0

X (x – 6) + 2 (x – 6) = 0

(x – 6) (x + 2) = 0

X =6 or -2

1. Find the values of x and y such that Y = $^{1}/\_{2}$(x2 - 3) and x + y = 6.

**Solution;**

Y = $^{1}/\_{2}$(x2 - 3) ------ (1)

 x + y = 6 ----------- (2)

From (2) y = 6 – x

Substitute y value into (1)

Y = $^{1}/\_{2}$(x2 - 3) will become

6 – x = $^{1}/\_{2}$(x2 – 3)

Multiply through by 2 to clear fraction

2 (6 – x) = x2 – 3

12 – 2x = x2 – 3

i.e. x2 + 2x – 15 = 0

Factorizing,

X2 + 5x – 3x – 15 = 0

X(x + 5) – 3(x + 5) = 0

(x + 5) (x – 3)=0

X+5=0 or x-3=0

X=-5 or 3

Substitute x value into (2)

X + y = 6 will become

-5 + y = 6 or 3 + y = 6

Y=6+5 or y =6 – 3

Y=11 or 3

The solution set are ( -5, 11) and (3,3)in (x,y) format

**ASSIGNMENT**

1. Write down the equation whose roots are the point of intersection of the graphs of Y = X2 + x – 2 and Y = x + 1
2. Solve the equation, p + q = 3 and p2 – q2 = 15