**Simple Linear Equation In Modulo Arithmetic.**

Sometimes simple linear equations can be in a given base. We solve them in accordance with all that we learnt so far, making use of our knowledge of how to solve simple linear equation.

Examples: from each of the following find the value of x.

1. 3x $⊕$ 2 = 5 (modulo 8)
2. 3 $⊖$ 5x = 11 (modulo 4)
3. 2x $⊕$ 13 = 8 (modulo10)
4. 5x $⊖$ 6 = 7 (modulo 8)
5. 2x $⊕$ 7 = 2 (modulo 9)

Solutions:

1. 3x $⊕$ 2 = 5 (modulo 8) taking $⊕$ 2 to the right hand side of the equation.

3x = 5 $⊖$ 2 (modulo 8)

3x = 3 (modulo 8) … dividing both sides by coefficient x (3)

$\frac{3x}{3}$ = $\frac{3}{3}$ (modulo 8)

X = 1 (modulo 8).

1. 3 $⊖$ 5x = 11 (modulo 4)

5x = 3 – 11 (modulo 4) since cannot subtract 11 from 3 we simplify by continuing adding 4 to 3 till we get first integer that is bigger than 11

5x = (3 $⊕$ 4 $⊕$ 4) – 11 (modulo 4)

5x = 11 – 11 (modulo 4)

5x = 0 (modulo 4) dividing both side by 5

$\frac{5x}{5}$ = $\frac{0}{5}$ (modulo 4)

X = 0 (modulo 4)

1. 2x $⊕$ 13 = 8 (modulo10)

2x = 8 – 13 (modulo 10) by simplifying

2x = (8 $⊕$ 10) – 13 (modulo 10)

2x = 18 – 13 (modulo 10)

2x = 5 (modulo 10) divide both sides by 2

$\frac{2x}{2}$ = $\frac{5}{2}$ (modulo 10)

X = $\frac{5}{2}$ (modulo 10)

This is insoluble since 2 cannot divide 5 even if we simplify by adding 10.

Therefore, the answer to the question [2x $⊕$ 13 = 8 (modulo10)] is insoluble.

Solve question 4 and 5.

 **INDICES**

This is the method of shorting a product of the same number or equal factor. For example a x a x a x a can be written as a4, where the number “4” indicates the number of factors multiplied together. Also from the above example “a” is called the base and 4 is called the index or exponent or power.

**5a4**

Index or exponent or power

base

coefficient

**LAWS OF INDICES**

1. Multiplication law: This law is obeyed when the bases are equal. Once the bases are equal we take one of the base and add the index. For example am x an  = am+n. But if the bases are not the same, we simplify them one after the other and multiply the out come. Example, 23 x 31 = 23 = 8 and 31 = 3. Therefore, 8 x 3 = 24.
2. Division law: This law is also obeyed when the bases are equal or the same. Once they are equal, we take one of the base and subtract the index. Example am ÷ an = am-n.

74 ÷ 73 = 74-3 = 71 = 7.

1. Zero index law: Any number raise to the power zero (0) is equal to unity (one)

Example: a0 = 1. 1000000 =1

1. Negative index law: this law states that a-m = $\frac{1}{a^{m}}$
2. Fractional index law: this law states that $a^{\frac{m}{n}}$ = $(\sqrt[n]{a} )$m. eg. $27^{\frac{2}{3}}$ = $(\sqrt[3]{27} )$2 = 32 = 9.
3. Multiple index law: this states tha (am)n = amxn = amn.

Other examples: Evaluate each of the following.

1. 36 x 3-4
2. 29 ÷ 2-1
3. 45 x 4-5
4. 6-2
5. $(0.25)^{-\frac{1}{2}}$
6. ($\frac{1}{2}$)5 x $(0.5)^{1\_{2}^{1}}$
7. $(32)$0.2

Solutions:

1. 36 x 3-4 using multiplication law

36+(-)4 = 36-4

32

1. 29 ÷ 2-1

29 – (-) 1 = 29+1

210

1. 45 x 4-5

45+(-)5 = 45-5

40  = 1

1. 6-2

$\frac{1}{ 6^{2}}$ = $\frac{1}{36}$

1. $(0.25)^{-\frac{1}{2}}$ First convert 0.25 to fraction (0.25 = $\frac{25}{100}$)

$\left(\frac{25}{100}\right)^{-\frac{1}{2}}$ Applying negative index law

$(\frac{\frac{1}{25}}{100})^{\frac{1}{2}}$ applying fractional index law we have

$(\sqrt{\frac{\frac{1}{25}}{100}})^{1}$ taking the square root we have

$\frac{\frac{1}{5}}{10}$ = $\frac{1}{1}$ x $\frac{10}{5}$

$\frac{10}{5}$ = 2 ans

1. ($\frac{1}{2}$)5 x $(0.5)^{1\_{2}^{1}}$ first convert $(0.5)^{1\_{2}^{1}}$ to fraction ($\frac{5}{10})^{\frac{3}{2}}$ and dividing out ($\frac{1}{2})^{\frac{3}{2}}$

($\frac{1}{2}$)5 x ($\frac{1}{2})^{\frac{3}{2}}$ convert to negative index law [($\frac{1}{2}$)5 = 2-5 and ($\frac{1}{2})^{\frac{3}{2}}$ = ($2)^{-\frac{3}{2}}$}

2-5 x ($2)^{-\frac{3}{2}}$ applying multiplication law

$2^{-5+(-\frac{3}{2})}$ Solving this we have

$2^{-\frac{10-3}{2}}$

$2^{-\frac{13}{2}}$ ans in index form.

Follow the above examples and solve example 7.

Application of indices to simple indicial equation:

Methods

1. Convert sides to be the same base
2. Equate the indices on the left to the one at thee right
3. Solve the resulting equation.

Examples: solve the following equations for x.

1. 32x = 2
2. 4x = 8
3. 5-x = 25
4. 22x – 5(2x) + 4 = 0
5. 2x+y = 8, 3x-y = 1 find the value of x and y
6. 27x+2 = 92x
7. 52x – 26(5x) + 25 = 0
8. 22x + 2x+1 – 8 = 0
9. $\frac{27^{x}}{9^{x+2}}$ = 81
10. $4$2x+y = 16, 5x-y = 25 find the value of x and y

Solutions: we are going apply the above methods in solving the above examples:

1. 32x = 2 converting both sides to the same base we have (32x = 2x(5) and 2 = 21)

25x = 21 equating the index on the left to the one at right we have (5x = 1)

5x = 1 divide both sides by 5.

$\frac{5x}{5}= \frac{1}{5}$

X = $\frac{1}{5}$ ans

1. 4x = 8 following the above steps we have

22x = 23

2x = 3

$\frac{2x}{2}= \frac{3}{2}$

X = $\frac{3}{2}$ or 1.5 ans

1. 5-x = 25 following the above steps as well we have

5-x = 55

-x = 5 divide both sides by -1

$\frac{-x}{-1}= \frac{5}{-1}$

$x= -$5

1. 22x – 5(2x) + 4 = 0 first thing to do is to convert 22x to multiple index law (22x = 2x(2))

2x(2) – 5(2x) + 4 = 0 let 2x be x that is any place we see 2x we call it x

x(2) – 5(x) + 4 =0

x2 – 5x + 4 = 0 this has result to quadratic equation so we are going to solve it using factorization method. Firstly, we are going to find two factors such that when we multiply them we get +4 and when we sum them we get -5. And the two factors are -1 and -4.

x2 – x – 4x + 4 = 0 by grouping the equation

(x2 – x) – (4x + 4) = 0 bringing factors that can factorize factors in the bracket which is x and 4 we

x(x2 – x) – 4(4x + 4) = 0

x(x – 1) – 4(x - 1) = 0

(x – 1) (x – 4) = 0 equating (x – 1) and (x – 4) to 0

x – 1 = 0 or x – 4 = 0

x = 0+1 or x = 0 + 4

x = 1 or x = 4

But recall that we said, ‘let 2x be x’. therefore, we are going to use this to find the value of x.

2x = 1 or 2x = 4 converting both sides to the same base

2x = 20 or 2x = 22 equating the indices we have

x = 0 or x = 2 ans.

1. 2x+y = 8, 3x-y = 1 firstly we are going to convert equations to the same base

2x+y = 23

3x-y = 30 secondly we are going to equate the indices

x + y = 3 ….. (1)

x – y = 0 ….. (2) this has result to simultaneous equation. We are going to solve it using elimination method by subtracting equation 2 from equation 1.

x + y – x – (-)y = 3 – 0 collecting like terms

x - x + y + y = 3 – 0

2y = 3 dividing both sides by 2

$\frac{2y}{2}= \frac{3}{2}$

y = $\frac{3}{2}$ putting this in equation 1 above we have

x + $\frac{3}{2}$ = 3 taking $\frac{3}{2}$ to the right hand of the equation we have

x = 3 - $\frac{3}{2}$

x = $\frac{6-3}{2}$

x = $\frac{3}{2}$

therefore, the value of x and y is $\frac{3}{2}$ and $\frac{3}{2}$ respectively.

Follow the above examples and solve example 6 to 10

**MEANING OF LOGARITHM OF NUMBERS.**

The logarithm of a number ‘n’ to base ‘a’ is the index of the power to which the base must be raised in order to give you the number.

The logarithm of ‘n’ to base ‘a’ is written as $log\_{a}n$. Thus, if ax = n, then x = $log\_{a}n$. Logarithm to base 10 are called common logarithm while logarithm to base e $≅2.718$ are called Natural logarithm and is written as $log\_{e}n$.

Parts to logarithm of a number.

There are two parts to logarithm of a number viz: Characteristics and Mantissa.

* Characteristics: This is the whole number part of the logarithm. For example the characteristics of the logarithm 99, 55.1 and 230 are 1, 1 and 2 respectively. This is gotten by converting 99, 55.1 and 230 to standard form. ie 99 = 9.9 x 101, 55.1 = 5.51 x 101 and 2.3 x 102. After finding the characteristics of the number the next thing is to find the mantissa.
* Mantissa: This is the fractional part of the logarithm of the number from the table. Usually the number to be found is in two digits but where it is a three digit number, we find the first two digits under the third digit. If it is a four digit number, we find the first two digit under the third digit and read the fourth digit from the difference table, which we add to the first reading.

Here are some examples.

1. In the case of two digit , find log 30. Here we find the characteristics which is 1 then find the mantissa from the mathematical table which is 30 under 0 = 4771. Therefore, log 30 = 1.4771
2. In the case of three digit, find log 723. We follow the same method above, then we can find the mantissa in the table, ie 72 under 3 which is 8591. And the characteristics is 2. Therefore, log 723 = 2.8591
3. In the case of four digit. Find log 2834.

The characteristics of 2834 is 3

We will find 28 under 3 which is 4518 and the difference of 4 which 6.

Finally we add 4518 and 6 which is 4524.

Therefore, log = 3.4524.

Finding the anti-logarithm of a number.

An antilogarithm of a number is the inverse of the logarithm of a number. To read the antilogarithm of a number from the table we follow the same method as reading a logarithm of a number. The only difference is that we first find the mantissa part then add characteristics part as decimal as in examples below.

Find the antilogarithm of the following.

1. 2.3300
2. 2.6410
3. 2.1628.

Solutions:

1. 2.3300 here we are going to find the antilog of 33 under 0 difference 0

2138 taking the decimal point using the characteristics which 2 we have

213.8 ans.

 2. 2.6410 here we find 64 under 1

 4375

 437.5

 3. 2.1628 here we find 16 under 2 difference 8

 1452 + 3

 1455 = 145.5 ans

**Use of the logarithm table and anti-log table to carryout operation.**

Before we can use logarithm table to carry out operations like multiplication, division and exponentiation we must first know the rule of logarithm.

Rules of logarithm.

1. Multiplication law: Multiplication law states that log of a product of a number equals the sum of the logs. $log\_{a}xy$ = $log\_{a}x$ x $log\_{a}y$ = $log\_{a}x$ + $log\_{a}y$
2. Division law: This states that log of quotients equals the difference of the log. $log\_{a}x÷y$ = $log\_{a}x$ - $log\_{a}y$
3. Power law: The log of a number raised to power is the product of the power and the log of the number. $log\_{a}x^{n}$ = $nlog\_{a}x$

Examples: using your logarithm table to evaluate the following.

1. $log\_{3}81$
2. $log\_{5}25$
3. 3.5 x 6.8
4. 389.6 ÷ 21.82
5. 7.832
6. $\sqrt[3]{961.9}$

Solutions

1. $log\_{3}81$ we are going to deduce this to index form by equating it to x ($log\_{3}81$ = x)

$log\_{3}81$ = x

3x  = 81

3x = 34

x = 4 ans.

1. $log\_{5}25$ we are going to follow the same process above

 $log\_{5}25$ = x

5x = 25

5x = 52

x = 2

questions 3 to 6 will be solved in a tabular form. and this involve the use of logarithm table and anti-log table. We first find the logarithm of the numbers then, apply the rules stated above before finding the anti-logarithm of number.

No log

3.5 0.5441

6.8 +0.8325

3.5x6.8 1.3766 anti-log of 1.3766 is 23.80

23.80

1. 3.5 x 6.8
2. 389.6 ÷ 21.82

No Log

389.6 2.5906

21.82 1.3389

389.6÷21.82 1.2517 anti-log of 1.33389 is 17.85

17.85

1. 7.832

No Log

7.83 0.8938 x2

7.832 1.7876

61.32

No Log

961.9 2.9831 (x3)

$\sqrt[3]{961.9}$ 0.9944

9.87

1. $\sqrt[3]{961.9}$

**SET THEORY**

A set is a collection of well defined objects.

Few points about sets:

1. Objects in a set are called elements.
2. Sets are denoted by capital letters A, B, C, and so on
3. If an element x is in a set A, we say that x belongs to A and it is shown as x $\in $ A.

Types of set.

There are many types of sets. These include:

1. Null or Empty set, $∅$ or {}: This is a set which dose not contain any element.
2. Equal sets: Two sets A and B are equal if and only if for every x belonging to A, x also belongs to B and for every x belonging to B, x also belong to A. for example, A = {2,3,6} B = {2,3,6}, then A = B.
3. Disjoint sets: Two sets A and B are said to be disjoint if they have no element in common. For example P = {1,3,5,7) and Q = {2,4,6} are disjoint because they have no element in common.
4. Subset: If for every element belonging to A also belongs to B then, we say that A is subset of B written as A $⊂$ B. for example if A = {1,2,3} and B = {1,2,3,4,5} then A $⊂$ B
5. Set of sets: If the elements of a set are also sets then, the set is called a set of set or class of set or family of set. For example, A = {{2,5},{5},{1,7,9},{0}} is a set of set.
6. Universal set U: This is a set that contains all the elements pertaining to a given problem, that is every other set being study is a subset of the universal set. For example, let A = {1,3,5}, B = {2,4,6}, C = {1,3,4,6}, D = {1,2,3,4} and U = {1,2,3,4,5,6,7,8,9}. Then U is a universal set to A,B,C,D.
7. Finite and Infinite Set: A set which has a finite number of elements is called finite set while if the number of elements in not finite then, the set is called infinite. For example, A = {x $\in $ N: 3≤x≤5} = {3,4,5} is a finite set because it has three elements while B = { x $\in $ N: x >5} = {6,7,8,9,10,11,……} is infinite because you cannot count the number of elements.

**OPERATION OF SET.**

There are different operations that can be performed on a set. These include:

1. Complement of a set: The complement of a set A denoted by $A^{´}$, for example is the set of all those elements that belong to the universal set U but dose not belong to set A.

For example, let U = {1,2,3,4,5,6,7,8,9}, A = {2,4,6,8}, B = {1,3,5,7,9}, C = {1,2}, D = {2,3,6,8} find $A^{´}$,$ B^{´}$,$ C^{´}$,$ D^{´}$.

Solution:

$A^{´}$ = U – A = {1,3,5,7,9}

$B^{´}$ = U – B = {2,4,6,8}

$C^{´}$ = U – C = {3,4,5,6,7,8,9}

$D^{´}$ = U – D {1,4,5,7,9}

1. Union of sets: The union of two sets A and B written as A$∪$B is the set of all elements that belong to A or belong to B without a repetition.

For example: let A = {2,3,6,8} and B = {1,2,3,4,7}, then find A$∪$B

Solution:

A$∪$B = {1,2,3,4,5,6,7,8}

1. Intersection of sets: The intersection of two sets A and B written as A $∩$ B is the set of all those elements that belong to A and also belong to B.

For example: if A = {a,b,c,d}, B = {c,d,e,f} find A $∩$ B

Solution:

A $∩$ B = {c,d}

1. Difference set: The difference set A – B is the set all elements that belong to A but dose not belong to B.

For example if A = {2,3,4,7,8} and B = {2,4}, find A – B

Solution:

A – B = {3,7,8}

More examples: Let the universal set be the set of integers from 1 to 15 and subsets A and B be defined in such a way that A is the set of even numbers and B the set of multiples of 3. Find

1. List the elements of the 3 sets above.
2. Find $A^{´}$ and $B^{´}$
3. Find A $∩$ B and A$∪$B
4. Find A$∩B^{´}$
5. Show that $(A∩B)^{´}$ = $A^{´}∪ B^{´}$

Solutions:

1. Elements of the 3 sets above are

The universal set which is integers from 1 to 15

U = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15}

Set A is even numbers that lies from 1 to 15

A = {2,4,6,8,10,12,14}

Set B is multiples of 3

B = {3,6,9,12,15}

1. Find $A^{´}$ and $B^{´}$ here we are required to find $A^{´}$ and $B^{´}$

$A^{´}$ = {1,3,5,7,9,11,14,15} and

$B^{´}$ = {1,2,4,5,7,8,10,11,13,14}

1. Find A $∩$ B and A$∪$B

A $∩$ B = {6,12}

A$∪$B = {2,3,4,6,8,9,10,12,13,14,15}

1. Find A$∩B^{´}$ here we are going to consider set A and B complement

A$∩B^{´}$ = {2,4,8,10,14}

1. Show that $(A∩B)^{´}$ = $A^{´}∪ B^{´}$ to do this we are going to value of $(A∩B)^{´}$ and the value of $A^{´}∪ B^{´}$ and compare our results.

A $∩$ B = {6,12} and

$(A∩B)^{´}$ = {1,2,3,4,5,7,8,9,10,11,13,14,15}

$A^{´}$ = {1,3,5,7,9,11,14,15} and

$B^{´}$ = {1,2,4,5,7,8,10,11,13,14}

$A^{´}∪ B^{´}$ = {1,2,3,4,5,7,8,9,10,11,13,14,15}

From the results we got above we can conclude that $(A∩B)^{´}$ = $A^{´}∪ B^{´}$

1. Venn-Euler Diagrams: Venn diagrams are diagrams used to representing sets. In this type of representation a square stands for the universal set. Subsets of the universal set are represented with circles. The intersection of subset is represented by any region that the sets share together.

A

B

U

$A∩B$ – Intersection of A and B

A$∪$B – Union of A and B

U

A

B

Examples on Venn diagram.

1. Out of 400 students in a final year in a senior secondary school, 300 are offering Biology and 190 are offering Chemistry.
2. How many students are offering both Biology and Chemistry, if only 70 students are offering neither Biology nor Chemistry?
3. How many students are offering at least one of the biology or chemistry?
4. In a class, 18 students repeated mathematics in an exam, 20 repeated English language in another exam, 7 repeated both and 5 did neither exam. How many students are in the class?
5. Of 158 TV viewers questioned, 94 watched NTA and 85 watched ABS. How many watched both?
6. In a certain hostel of 60 inmates to whom Akara and bread are served as breakfast, one morning 37 inmates took Akara and 40 took bread. How many students took Akara and bread?
7. At a certain social gathering of 72 guests during which Beer and Malt drink were served, 39 guests took Beer, 7 took both Beer and Malt and 4 none of the two drinks.
8. How many guests took Malt drink?
9. How many guests took malt drink only?

Solutions: if you are given any question involving Venn diagram, the first thing to do is to represent the information given to you in Venn diagram.

1. let x be the number that offer both subjects, B to represent biology and C to represent chemistry.

U(400)

300 – x

190 – x

70

x

B

C

1. To find the number of students that offer both subjects we sum 300 – x, x, 190 – x and 70 and equate it to 400.

300 – x + x + 190 – x + 70 = 400 … collect like terms

300 + 190 + 70 – x + x – x = 400

560 – x = 400

560 – 400 = x

160 = x

Number of students that offer both subjects is 160.

1. To find the number of students that offer at least one of biology or chemistry, we first find the number that offer only biology and the one that offer chemistry only and sum them together with number of those that offer both subjects.

Number that offer biology only = 300 – 160 = 140

Number that offer chemistry only = 190 – 160 = 30

Number that offer both = 160

Number that offer neither of them = 140 + 30 + 160 = 330 students ans.

1. To find the total number of student in the class we first find number of students that repeat mathematics only, the number that repeat English only and sum them together with those that repeat both and those that did neither of the two after representing the information in Venn diagram.

M

E

U(?)

5

18 – 7

20 – 7

7

Number that repeated mathematics only = 18 – 7 = 11

Number that repeated English only = 20 – 7 = 13

Total number of students in the class = 11 + 13 + 7 + 5 = 36 students ans.

Follow the process above and solve example 3 to 5.

**NOTE: Try and read all because it’s self explanatory note. But if there is anything you didn’t understand send your questions through this my email address (****jamaicaspecie@gmail.com****). Good luck.**